Inertia Effects on the Flow of Viscoplastic Fluids Through an Abrupt Contraction

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Abstract

This paper is concerned primarily with the effects of inertia (Reynolds number) on the laminar flow of a non-Newtonian viscoplastic fluid through a sudden contraction in the planar and axisymmetric geometries. The influence of Reynolds number on the structure of the flow and pressure losses is studied using a numerical approach. The radius of the rigid moving zone is always unchanged for the different Re, but the downstream rigid moving zone is pushed away and it will vanish at Re=303. Inertia and yield stress act in opposite ways. When Re increases, pressure loss due to the singularity decreases. On the contrary, as Bingham number increases, the pressure loss increases significantly. Moreover, a comparison between pressure drops in sudden expansion and abrupt contraction has been performed. The pressure loss due to the sudden expansion is higher than contraction (55%) until Bi= 100, in which they reach the same value of 2.8. Finally, the provided numerical results are validated by available experimental data.

Keywords: Yield-stress, Reynolds number, Herschel-Bulkley, Inertia effects, Un-yielded zones

Introduction

Viscoplastic fluids are a category of non-Newtonian fluids that is characterized by having a yield stress. In other words, below a certain critical shear stress there is no deformation of the fluid and it behaves like a solid, but when that yield value is exceeded, the material flows like a fluid. Paint, slurries, pastes, and food substances like margarine, mayonnaise, and ketchup are good examples of viscoplastic fluids [1]. Creeping flow of yield-stress fluids through an abrupt contraction has been mainly investigated in the planar and axisymmetric cases [2,3,4]. Investigations in contraction flows considering the inertial effects are less common but not unknown. Chaparian et al. [5] presented a comparison of viscoplastic and elastoviscoplastic flows in porous media considering the effects of inertia. Varges et al. [6] studied the steady-state flow of a non-Newtonian fluid in a planar channel with sudden expansion. They calculated the velocity and pressure fields for Re between 1 and 40. Vradis et al. [7] studied the inertia effects on the flow of Bingham plastics through sudden contractions in a pipe. They calculated velocity and vorticity fields for moderate and high Reynolds numbers (Re=10 and 1000). Burgos et al. [8] calculated the influence of inertia on structure of the flow in sudden expansion for Re varied between 0.01 and 100. Recently, Blanco [9] studied viscoplastic fluid flow of Carbopol solutions using experimental study, for laminar and turbulent regimes through an abrupt contraction. Thompson and Soares [10] analyzed the dimensionless numbers that concern the flow of viscoplastic materials. They emphasized on the importance of Re in simulation of yield-stress fluids.

The purpose of this paper is to investigate the influence of inertia ($0.001 \leq Re \leq 100$) on the structure of un-yielded zones and on the pressure losses due to the sudden contraction (contraction ratio= 4). The results are obtained for the both planar and axisymmetric geometries. Moreover, a comparison between pressure drop in sudden contraction and sudden expansion is provided. Finally, the numerical results are validated by available experimental information.

Methodology

A. Governing equations

The flow is modeled using the conservation of mass and momentum for an incompressible fluid [1]. For a viscoplastic fluid with τ_0 being the yield stress, the Herschel–Bulkley (HB) model can be applied. The main reasons to utilize this model are as flollows:

- The Herschel-Bulkley law is sensible because the material structure that resists deformation and leads to the yield stress is not completely destroyed at $\tau = \tau_0$.
- This model exhibits both a yield stress and a nonlinear viscosity (shear dependant viscosity).

To eliminate the discontinuity of the HB model, Papanastasiou's modification [11] is employed as

$$\tau = \left(K\dot{\gamma}^{n-1} + \frac{\tau_0[1-\exp(-M\dot{\gamma})]}{\dot{\gamma}}\right)\boldsymbol{\dot{\gamma}} \qquad (1)$$

Here, K and n are the consistency and the shear-thinning index, respectively; $\dot{\gamma}$ is the magnitude of strain rate tensor ($\boldsymbol{\dot{\gamma}}$). Value of n is set to 0.37. M is the regularization parameter and M = 1000 is chosen here.

Reynolds and Bingham numbers characterizing a Bingham fluid flow are defined, respectively, as

$$Re = \frac{\rho R_2 U_2}{K(\frac{U_2}{R_2})^{(n-1)}}, \qquad Bi = \frac{\tau_0}{K(\frac{U_2}{R_2})^{n}}, \qquad (2)$$

The Reynolds and Bingham numbers are formed by scaling velocities with the average velocity U_2 in the small channel and distances with the radius R_2 of the same channel.

Pressure losses were characterized by the equivalent entrance length, defined by

$$L_{eq} = \frac{\Delta p_s}{2\sigma_{w2}} = \frac{\Delta p - \Delta p_1 - \Delta p_2}{2\sigma_{w2}} \qquad (3)$$

Where Δp_s represents the additional pressure loss due to the singularity; ΔP is the total pressure drop; Δp_1 is the pressure drop in fully developed Poiseuille flow in a tube of length $L_i$and radius
R_i; Δp_2 is the wall shear stress in fully developed Poiseuille flow in a tube of radius R_2.

B. Numerical approach

The finite element software, COMSOL, is utilized for the simulation. This code is based on mixed pressure-velocity formulation, accepted algorithm for steady state viscous approximation. The resolution is based upon an iterative Newton scheme coupled with Picard iterations. Figure 1 shows the computational domain employed in this simulation and the mesh used throuout this paper. The mesh is highly refined in the corner of the expansion to capture the rigid static zones. The rest of the mesh corresponding to the main flow has larger elements in order to maintain a reasonable CPU time. The planar mesh finally chosen for this study has 7854 nodes and 3750 elements. The results are independent of the mesh.

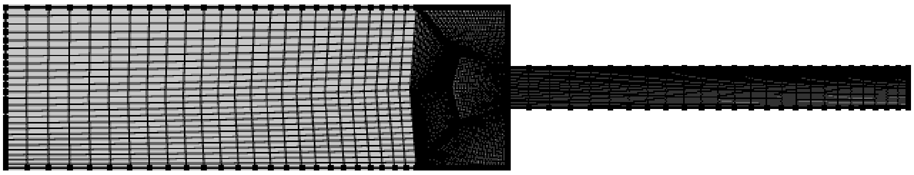


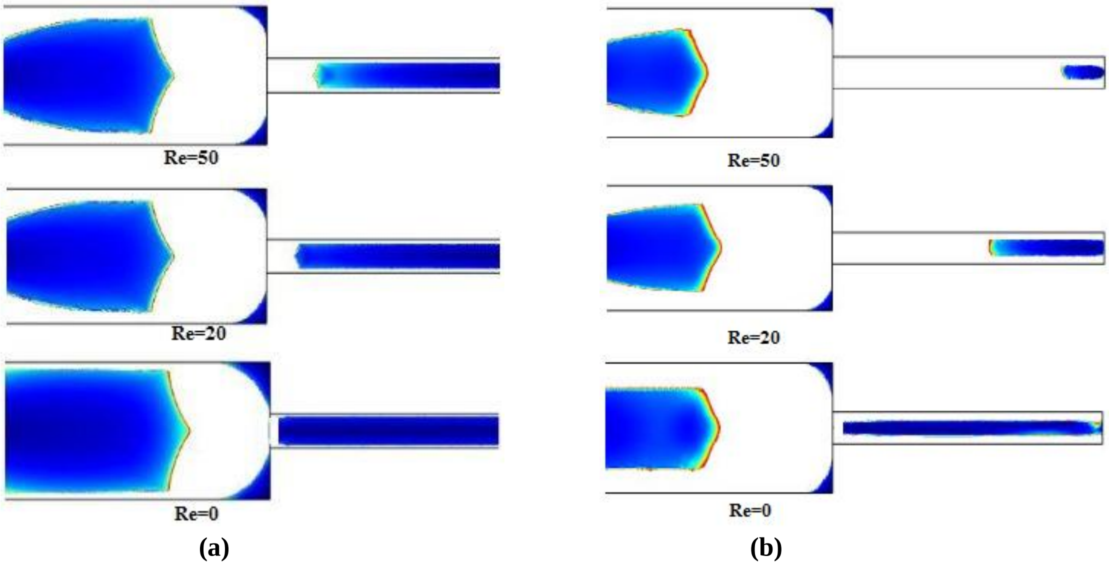
Fig. 1 Mesh used throughout this study

Fully developed flow is applied at entry and exit. The no-slip boundary condition is imposed at the walls. In all cases, the pressure is set to be zero at the exit of the geometry. For all the meshes, the length of the small channel is 20 times R_2 and the total length is 45 times R_2..

Results

A. Structure of the flow

The influence of Reynolds number on flow morphology is illustrated in figure 2.



(a) **(b)**

Fig. 2 Influence of inertia on structure of the flow in axisymmetic geometry for a) Bi=100 b) Bi=10

When the Reynolds number increases, the downstream rigid moving zone is pushed away. At Re=303, the downstream rigid moving zone is completely vanished. The radius of the rigid moving zone is always unchanged for the different Re values. On the contrary, as inertia increases, the rigid static zone decreases in size.

B. Influence of inertia on pressure loss

It is also interesting from the technological point of view to estimate the pressure and head losses due to the singularity [3]. The change in Leq (Eq. 3) versus Reynolds number is illustrated in the figures 3 and 4 for axisymmetric and planar models, respectively. For Re numbers smaller than 1, that is, when inertia effects are weak, pressure loss is independent of Re and only dependent on Bi. For larger Re numbers, Leq decreases with increasing Re and becomes a gain instead of a loss of pressure. This is owing to the fact that by accumulation of the vortex in the corner of the expansion, the flow slows down. As the kinetic energy decreases, the pressure must increase to ensure the energy conservation. For all Reynolds numbers, it can also be found that pressure loss increases with Bi.

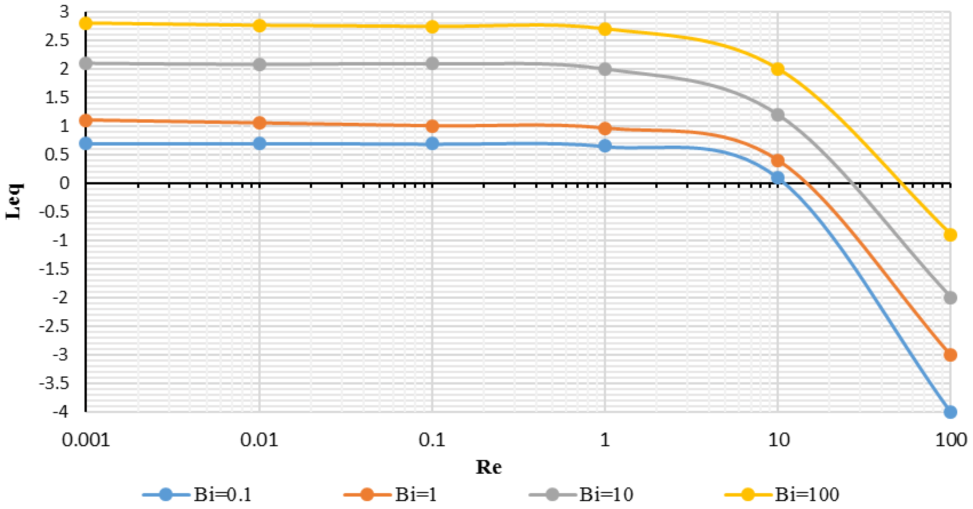


Fig. 3 Pressure drop versus the Reynolds number for different Bi (axisymmetric geometry)

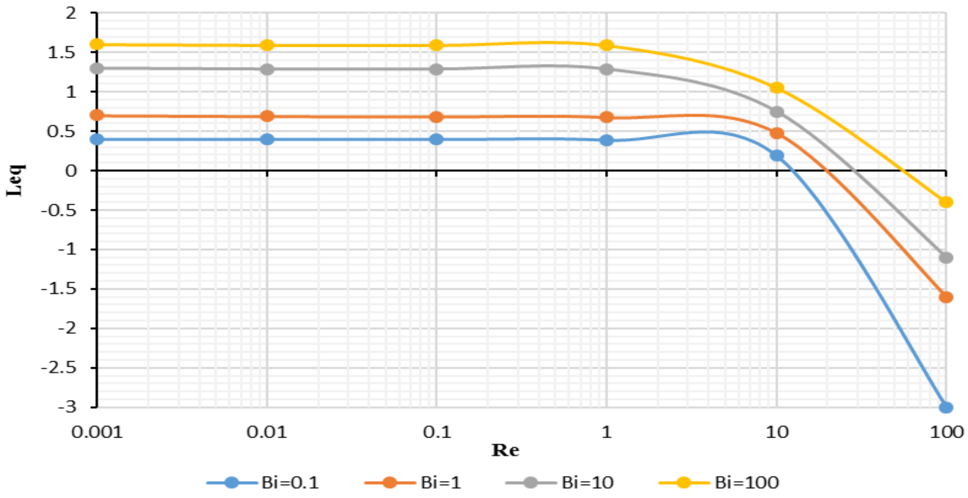


Fig. 4 Pressure drop versus the Reynolds number in planar (2D) geometry

In the planar case, the pressure loss due to the abrupt contraction is decreasing function of the Reynolds number (see figure 4). The pressure drop values in planar model is lower than in axisymmetric geometry (between 50-60% lower).

C. Comparison of sudden expansion and sudden contraction

Excess pressure drop results for an axisymmetric contraction and an axisymmetric expansion geometries are compared in the figure 5. The simulation is performed for creeping flow condition (negligible inertia) and for Bi numbers varied between 0.01 to 100. Pressure drops due to the sudden expansion is higher than those in sudden contraction. At Bi=0.01, there is the maximum difference (55%) between the pressure drops. As Bi increases, the difference between pressure drops decreases and finally at Bi=100, they reach the same value (2.8). This is due to the fact that at Bi=100, the size of the rigid static zone becomes the same in both contraction and expansion, and consequently the corresponding pressure drop in both geometries do not differ.

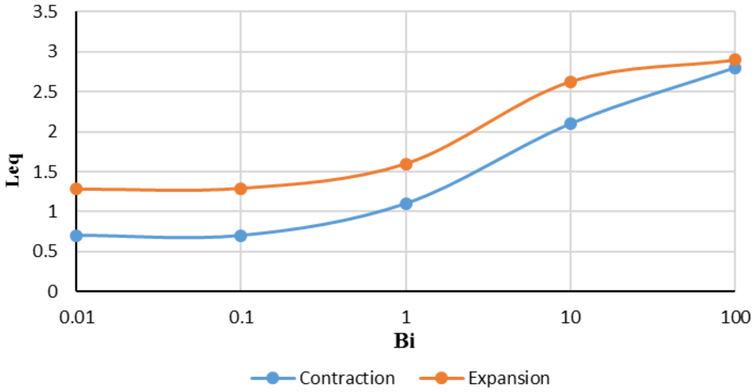
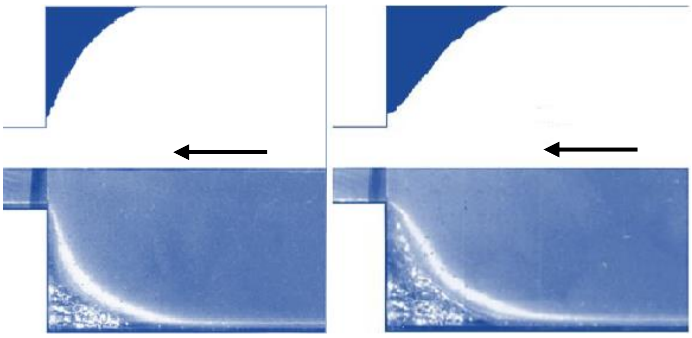


Fig. 5 Pressure drop comparison between sudden expansion and sudden contraction (Re=0.001, axisymmetric geometry)

D. Comparisons with experimental results

We have compared our numerical results with experimental ones obtained with a corbopol gel, the rheometric properties of which fit well with the Herschel-Bulkley model [9]. There is a relatively good agreement between the shape of the rigid static zones obtained experimentally and numerically. Nevertheless, the numerical approach seems to overestimate the dimension slightly (about 10–12% greater in the numerical case than in the experimental one)



(a) **(b)**

Fig. 6 Comparison of numerical and experimental results. (a) Re=10, Bi=100 (b) Re=0.001, Bi=100

Conclusions

The inertial flow of Hershel-Bulkley fluids through a sudden contraction (4:1) has been studied numerically. Influence of Re on the formation of un-yielded zones and the pressure drop was presented. A summary of the results are as follows:

- Inertia and yield stress act in opposite ways. When inertia increases, the rigid dead zone decreases. Conversely, when yield stress increases, the rigid dead zone increases.

- Below Re=1, Leq is nearly constant. Beyond this value, pressure loss decreases with Re and becomes a gain instead of a loss of pressure.
- At low Bi ($Bi< 0.1$), the pressure drop in the sudden expansion is 55% higher than pressure loss in contraction. However, at higher Bi numbers ($Bi>100$), there is no difference between pressure loss due to the sudden contraction and sudden expansion.
- There is a very good agreement between the shape of the rigid static zones obtained experimentally and numerically. Nonetheless, the numerical method overestimates the size of the un-yielded zones (about 10–12%)

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