Viscoplastic Fluid Flows Through an Abrupt Contraction

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Abstract
Numerical simulations were performed for the creeping flow of a yield-stress fluid through a sudden contraction using the Herschel-Bulkley model with a Papanastasiou's modification. The influence of yield-stress and contraction ratio values on the structure of the flow was investigated. All the simulations were carried out for 2D, 3D, and axisymmetric geometries. The detailed structure of flow shows two rigid zones, one located at the corner of the contraction (rigid static zone) and one located close to the center-line of the channel (rigid moving zone). When yield stress increases, the size of the rigid dead zones increases significantly. Moreover, the pressure losses are given with yield stress, contraction ratio, and geometry. It is seen that 2D and axisymmetric models are not accurate indicating the need to account for the cross-section shape. Finally, the numerical results are compared with the available numerical and experimental data.

Keywords: Yield-stress, Bingham number, Herschel-Bulkley, 3D models, Un-yielded zones

Introduction
The flow of a viscoplastic fluid through contractions is distinguished from other non-Newtonian fluids by the existence of a plug region due to shear stress values smaller than the yield stress. [see figure 1]. Also, it is marked by the appearance of unyielded regions at the corners of the contraction. Moreover, in some cases, both conditions coexist as represented in Figure 1. From a technological point of view, for example to design an industrial process, it is very important to know the structure of the flow and how it changes in accordance with the governing parameters. In some industrial cases, such as food and pharmaceutical products, it is necessary to eliminate rigid static zones or reduce the size of the vortex.
Balmforth et al. [1], de Bruyn et al. [2] and Frigaard [3] discuss recent developments in this field of research. In the planar geometry, Link et al. [4] investigated the flow of viscoplastic fluids through a one-to-four sudden expansion using a finite element method. They calculated the un-yielded regions for different Bingham numbers varied between 0.04 and 20. Abdali et al. [5] studied entry and exit flows through extrusion dies using Bingham-Papanastasiou model

and determined the yielded zones for Bingham numbers varied between 1 and 2.7. Letelier et al. [6,7] demonstrated an analytical method to solve the viscoplastic fluid flow in the non-

circular tubes. Blanco [8] studied viscoplastic fluid flow of Carbopol solutions using experimental study, for laminar and turbulent regime through an abrupt contraction. Jay et al. [9] investigated the creeping flow ($Re \approx 0$) of a Herschel-Bulkley fluid through an axisymmetric expansion. Kfuri et al. [10] studied the flow development of Hershel-Bulkley fluids in a 3D square expansion, in which they calculated velocity field and volume flow rate in relation to the pressure drop.
The purpose of this paper is to investigate the influence of contraction ratio and yield-stress (Bi number) on the structure of un-yielded zones and pressure losses in the creeping flow of a viscoplastic fluid through a sudden contraction. The main novelty of this paper is demonstration of flow structure and pressure losses in relation to 3D geometries, which is absent in all the papers available.

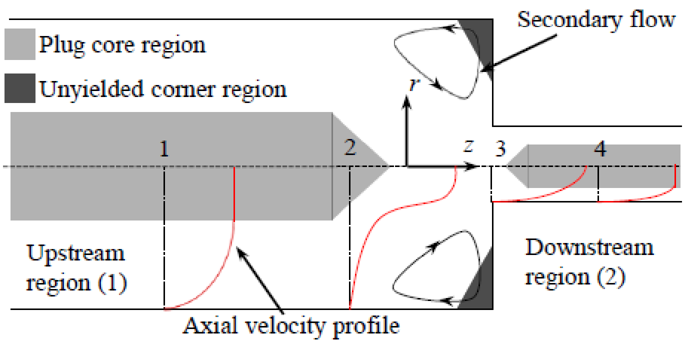

Fig. 1 Flow of viscoplastic fluid through a contraction. Points 1, 2, 3, and 4 presents the velocity profile changes along the contraction

Methodology

A. Governing equations

A fluid flow is governed by a continuity equation and a momentum equation as follows:

$$\nabla \cdot u = 0, \qquad \rho\left(\frac{\partial u}{\partial t} + u \cdot \nabla u\right) = \rho f + \nabla \cdot \sigma, \tag{1}$$

Where u and f are velocity and body force vectors, respectively; ρ is the fluid density; σ is the total stress tensor with p being the pressure, I the unit tensor, τ the deviatoric stress tensor, and $\dot{\gamma}$ the deformation rate tensor. For a yield stress fluid with τ_0 being the yield stress, the Herschel–Bulkley (HB) model can be employed [5]. To tackle the discontinuity of the HB or Bingham model, Papanastasiou's modification [5] is utilized as

$$\tau = \left(K\dot{\gamma}^{n-1} + \frac{\tau_0[1-\exp(-M\dot{\gamma})]}{\dot{\gamma}}\right)\dot{\boldsymbol{\gamma}} \tag{2}$$

Here, K and n are the consistency and the shear-thinning index, respectively; $\dot{\gamma}$ is the magnitude of strain rate tensor ($\dot{\boldsymbol{\gamma}}$). When $n = 1$, this model becomes a Bingham one. In this paper, n is
set to 0.37. M is the regularization parameter. For the sake of computational efficiency, M = 1000 is chosen here. Details about this can be found in the work of Abdali et al. [5]. The Herschel-Bulkley law is realistic because the material structure that resists deformation and leads to the yield stress is typically not completely destroyed at $\tau = \tau_0$. Instead, the structure

persists postyield and renders the viscosity shear-rate dependent. In other words, viscoplastic fluids usually exhibit both a yield stress and a nonlinear viscosity. The working fluid of this research, a Carbopol gel, is described relatively well by the Herschel-Bulkley model [1].
Dimensionless pressure drop and Bingham number characterizing a Bingham fluid flow are defined as

$$L_{eq} = \frac{\Delta p_s}{2\sigma_{w2}} = \frac{\Delta p - \Delta p_1 - \Delta p_2}{2\sigma_{w2}}, \qquad Bi = \frac{\tau_0}{K(\frac{U_2}{R_2})^n}, \tag{3}$$

Δp_s represents the additional pressure loss due to the singularity. where ΔP is the total pressure drop; Δp_1 is the pressure drop in fully developed Poiseuille flow in a tube of length $L_i$and radius R_i; Δp_2 is the wall shear stress in fully developed Poiseuille flow in a tube of radius R_2. The Bingham number is formed by scaling velocities with the average velocity U_2 in the small channel and distances with the radius R_2 of the same channel.

B. Numerical approach
COMSOL software is utilized for the computation. Figure 2 shows the computational domain employed in this simulation and the mesh used throuout this study. In order to obtain accurate results for the rigid dead zones, the mesh is highly refined in the corner of the expansion. The rest of the mesh corresponding to the main flow has larger elements in order to maintain a reasonable CPU time. The optimum mesh finally chosen for this study has 2738 nodes and 1350 elements (for the axisymmetric model). The results are independent of the mesh.

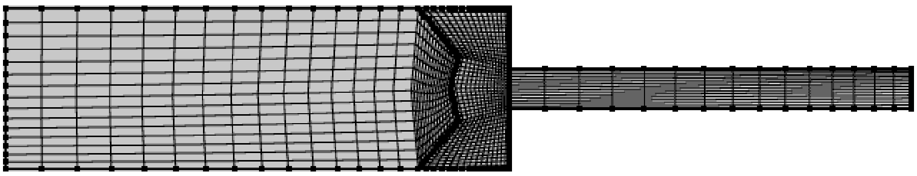

Fig. 2 Mesh used throughout this study

For all the meshes, the length of the small channel is 20 times R_2 and the total length is 45 times R_2. With these lengths, the fully developed conditions upstream and downstream are verified. The no-slip boundary condition is imposed at the walls.

Results
A. Structure of the flow
A.1. Influence of the Contraction Ratio
The influence of the contraction ratio on flow morphology was investigated for Bi=10. When the contraction ratio increases (from CR=2 to CR=8), the size of the rigid static zone in the corner also increases (see figure 3). On the other hand, the size of the downstream rigid moving

zone decreases and the extremity of the downstream rigid zone is pushed away. The extremity of the upstream rigid moving zone is pushed back. In all the cases, rigid zones take up a large part (about 70%) of the flow.

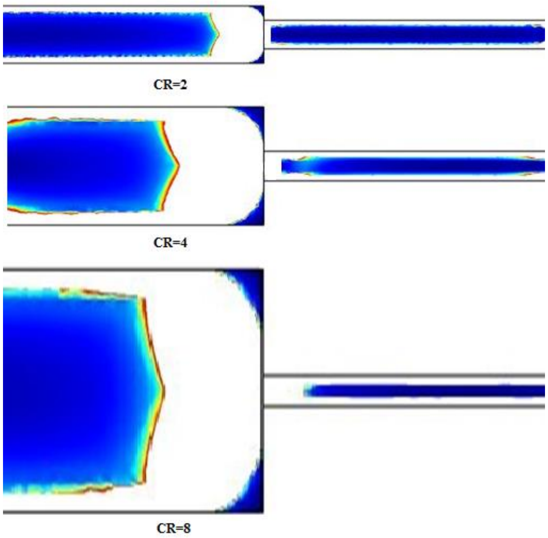

Fig. 3 Influence of the contraction ratio on the structure of the flow for Bi=10 in the axisymmetic geometry

A.2. Influence of the Bingham Number

The static region at the corner and the rigid moving zones in both small and large tubes are expanded drastically by increasing the Bi number (see figure 4).

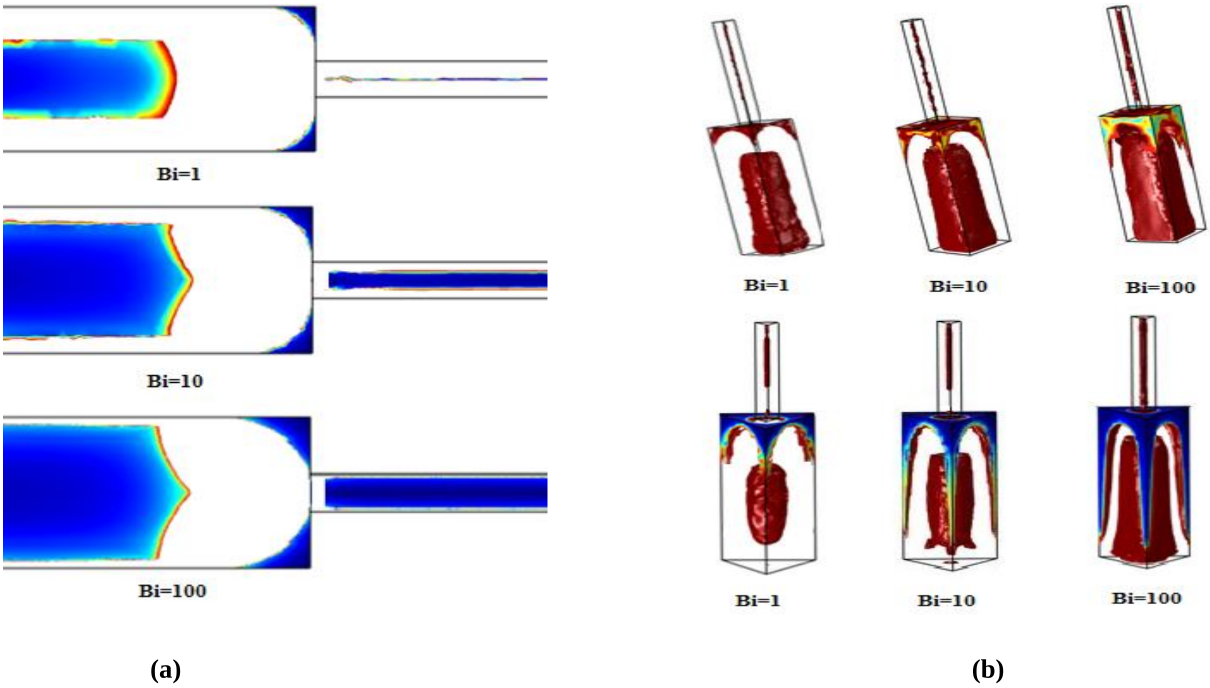

Fig. 4 (a) Influence of Bi numbers on the overall unyielded zones in the flow of the Hershel-Bulkley fluid in axisymmetric geometry (4:1) (b) Structure of the flow in 3D geometry (triangular & square cross sections)

For Bi=100, solid zones take up a large part of the flow both upstream and downstream (see figure 4(a)). Figure 4(b) shows the results for 3D geometries with square cross sections. In both 3D geometries, the solid moving zones are virtually identical, but the rigid static zones (at the corner of the contraction) are larger in the tube with triangular cross section. The moving rigid zones in the small tubes are very thin in comparison with the axisymmetric geometry.

B. Pressure drop

The change in Leq (see Eq. 3) versus contraction ratio is illustrated in figure 5. Leq is an increasing function of the contraction ratio. For a contraction ratio 8:1, a value of 4 is almost obtained for a Bi number equal to 100 (see figure 5).

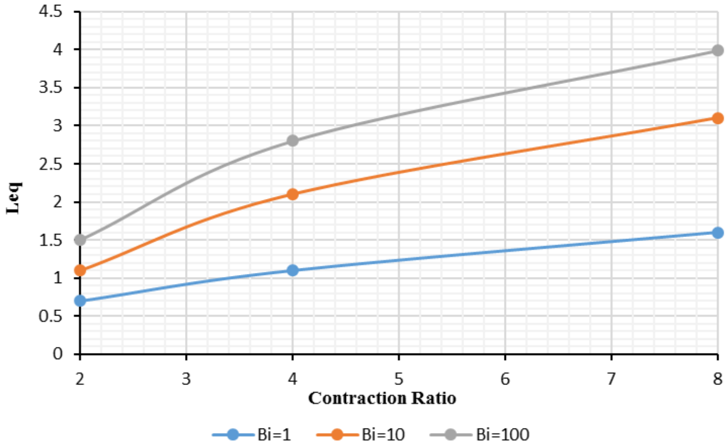

Fig. 5 Pressure drop versus the contraction ratio (axisymmetric geometry)

Figure 6 shows the pressure drop versus the Bingham number for different geometries. Below the Bi = 0.1, Leq is nearly constant. At this point, the rigid central zone begins to grow and therefore so do the pressure losses. These results were also compared with those obtained by Blanco [8] experimentally for axisymmetric model and by Abdali [5] for planar geometry. The results are very close (with a difference of between 0% and 12.5%). Pressure loss in 3D channel

with triangular cross section is more than others. These results confirm the importance of 3D simulation indicating the need to account for the cross-section shape.

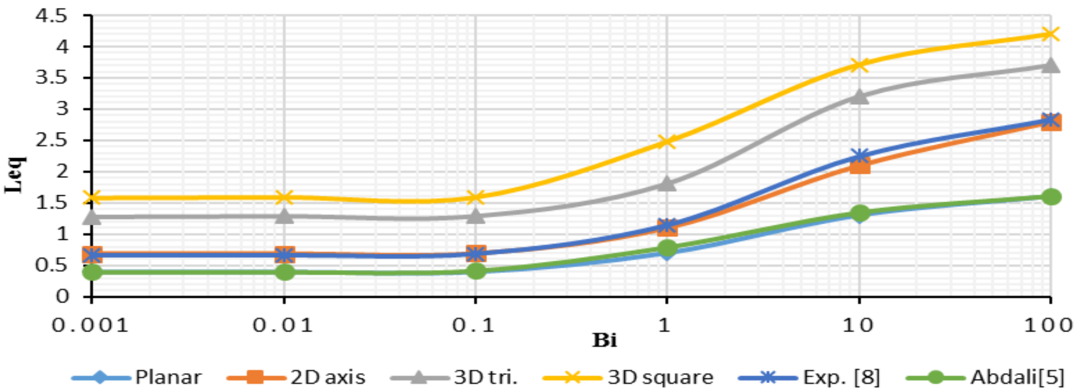

Fig. 6 Pressure loss versus Bi for different geometries and comparison with experimental and numerical data available (the contraction ratio is fixed 4:1 for all the cases)

Conclusions

Numerical simulations were carried out concerning the creeping flow of viscoplastic Herschel-Bulkley model through a sudden contraction. The detailed structure of the flow was studied for 2D, 3D, and axisymmetric geometries. In the corner of the contraction, a rigid dead zone can be found. There is also a rigid moving zone in the central part of the flow in both the upstream and downstream pipes. The influence of the governing parameters, that is, the Bingham number, contraction ratio, and geometry on this structure is identified.

Moreover, the pressure loss due to the singularity is calculated. A summary of the results is listed here:

- Structure of the flow highly depends on the Bi number. As Bi increases, the rigid zones increase drastically. Furthermore, increasing the contraction ratio results in the expansion of rigid moving and rigid static zones.
- Pressure loss is almost constant while the Bi number is smaller than 0.1. On the other hand, for greater values, the increase in Leq is very significant.
- These results show the importance of 3D simulation and how they differ from 2D and axisymmetric ones. This simplification (not considering the cross-section shape) will cause a large error in pressure loss calculations.

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