



A new simple empirical model for prediction of the viscosity of nanofluids

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Abstract

In this article, a new simple empirical model is proposed for the prediction of the viscosity of nanofluids which is only a function of volume fraction of nanoparticles. The new model is validated with a high number of experimental data of nanofluids containing water as the base fluid at the different types of nanoparticles, temperatures, and concentrations of nanoparticles available in the literature. The nanofluids contain Al₂O₃, SiO₂, TiO₂, GQD (graphene quantum dot), and CuO nanoparticles. Results of the new model were compared with experimental data and five other empirical models available in the literature, and absolute average relative deviation (AARD) between experiment and the results of the new model was obtained as 22.02 % which is acceptable and better than the previously reported models.

Keywords: Nanofluids, Empirical model, Volume fraction, Viscosity, Brinkman equation.

Introduction

Nanofluids are suspensions of metallic nanoparticles in conventional fluids that possess better thermal performance than single-phase fluids (water, ethylene glycol, and water-ethylene glycol). Nanofluids, with particle diameters, generally lying within 100 nm, have gained significant importance during the past two decades by scientific because of their properties such as enhanced heat transfer and CO₂ gas capture capabilities. The term nanofluid was created in 1995 by Choi [1], who presented that the uniform dispersion of low concentrations of nanoparticles into a typical liquid.

One of the essential properties of the fluid is viscosity. This property is greatly affected by the presence of solid particles in the liquid phase. Also, viscosity has a noteworthy role in determining the pumping power necessary for any industrial equipment. Therefore knowing the behavior of nanofluid viscosity is very significant in the design of industrial systems such as cooling and heating systems, pumping, and process equipment. Many of the researchers are attempted to measure experimentally the viscosity of the nanofluids such as Wang et al. [2] which measured the relative viscosity of Al₂O₃-water and Al₂O₃-ethylene glycol nanofluids, Sedaghat and Yousefi [3] studied the addition of the GQD nanoparticles to water, ethylene glycol, and ethylene glycol and Zhou et al. [4] investigated the



effect of sodium dodecyl sulfonate (SDS) and titania (TiO₂) nanoparticles on the viscosity of water. However, the measurement of viscosity is time-consuming and expensive. Therefore several models have been developed to describe the viscosity of the nanofluids. In original work, Einstein [5] reported a novel linear model for predicting the viscosity of suspensions of uniform spherical particles. Afterward, Brinkman [6], Batchelor [7] and Thomas et al. [8], investigated the interaction between particles in suspension and generalized the Einstein's work for use with moderate particle concentrations. In the same field, Mooney [9], and Metzner [10] independently proposed some other models for estimating the viscosity of nanofluids. Some of these presented models are simple and only a function of volume fraction of the nanofluids such as Einstein [5], Brinkman [6], Batchelor [7], Thomas et al. [8], etc. On the other hand, some models are complex and in addition to volume dependence, are also a function of the nanoparticle diameter, the density of the base fluid, and other characteristics of base fluid and nanoparticles. As examples of these type of models are the relationships presented by Koo et al. [11], Masoumi et al. [12].

The simple models underestimate the dynamic viscosity of the nanofluid even for relatively low particle concentration and models with acceptable predictions are complex and require many parameters. Also, these models demonstrated large discrepancies among each other which significantly restricting their applicability and are obtained from a small number of experimental data which severely limits the range of their validity. Therefore, in this study, a simple empirical model is proposed which is only a function of the volume fraction of the nanoparticle. Also, a comprehensive experimental data bank with 332 data points obtained from the literature is applied to validate the proposed model.

Modeling

The new simple model for prediction of the nanofluid dynamic viscosity is proposed as following,

$$\frac{\mu}{\mu'} = \frac{1}{1 - 5.88 * \varphi^{0.882} * e^{(0.762\varphi)}} \quad (1)$$

where μ' is the viscosity of the base fluid, μ is the viscosity of the nanofluid and φ is the volume fraction of the nanoparticle. The best constant parameters of this model (-5.88, 0.882, and 0.762) derivate from comprehensive experimental data available in the literature that their information is listed in table 1. This data bank consists of the five different nanofluids at various temperatures, the diameter of nanoparticles, and concentrations of nanoparticle and water as the base fluid.

The deviations between the results of the model and experimental data are considered in the form of AARD% (percent of average absolute relative deviation) as follows:

$$AARD\% = \frac{100}{N} \sum_i \left| \frac{\mu^{\text{exp.}} - \mu^{\text{cal.}}}{\mu^{\text{exp.}}} \right| \quad (2)$$

where, $\mu^{\text{exp.}}$ denotes experimental viscosity and $\mu^{\text{cal.}}$ shows the viscosity calculated by the models and N displays the number of the experimental data.



Table 1. Information about the systems considered in this work

Number	System	Temperature/K	Volume Fraction	Number of data	Ref.
1	water-Al ₂ O ₃	293.082-345.4158	0.0005-0.094	169	[13-15]
2	water-SiO ₂	292.9504-345	0.0011-0.0045	27	[13,14]
3	water-TiO ₂	305-345	0.0019-0.031	12	[13]
4	water-GQD	283.3598-313.2839	0.0004-0.005	54	[3]
5	water-CuO	294.8087-339.3038	0.01-0.09	70	[16]

Results and discussion

In this article, by using the new model introduced as equation (1) the viscosity of five different nanofluids at various conditions is predicted. To calculate the viscosity of the base fluid (water) at different temperatures, experimental data were collected from IAPWS 2008 and the following correlation is fitted with a coefficient of determination (R^2) equal to 0.9992:

$$\mu' = 3.7781 \times 10^{-6} \times (T / K)^3 + 3.7939962 \times 10^{-3} \times (T / K)^2 + 1.276293426 \times (T / K) + 144.2873267682 \quad (3)$$

where μ' is the viscosity of the base fluid and T is the temperature (K).

Table 2 shows the deviation between experimental viscosity and that calculated by the new presented model for five different systems and compared this deviation with the other previous model presented by Einstein [5], Brinkman [6], Batchelor [7], Tseng and Chen [17] and Thomas et al. [8].

Table 2. The percent of average absolute relative deviation (AARD%) of the presented model in this work and comparisons with the performance of earlier models.

Number	System	This work	Einstein [6]	Batchelor [7]	Brinkman [8]	Tseng and Chen [25]	Thomas et al. [10]
1	water-Al ₂ O ₃	20.25	37.98	37.36	47.07	71.90	37.47
2	water-SiO ₂	50.76	52.10	52.09	52.80	77.87	52.09
3	water-TiO ₂	52.13	55.46	55.42	57.19	80.18	55.43
4	water-GQD	3.79	3.79	3.79	4.23	55.03	3.79
5	water-CuO	24.12	46.09	45.59	53.11	76.60	45.68
Overall		22.02	35.91	24.51	42.21	70.93	35.56

As can be seen from Table 2, the results of the models show that the present model in this work demonstrates the best results with respect to the other models. Also, all six models have the most deviation for the water- TiO₂ solution, and the water-GQD solution has the least deviation. Fig. 1 shows the results of prediction of the viscosity by the presented model in this work at 283, 288, 293, 298, 303, 308, and 313 K and a wide range of GQD concentrations for the water-GQD system. In Fig. 1, the experimental data and calculated results are shown with points and solid lines, respectively. As one can observe, the correlated viscosity are in very good agreement with the experiments for all temperature and concentration ranges.

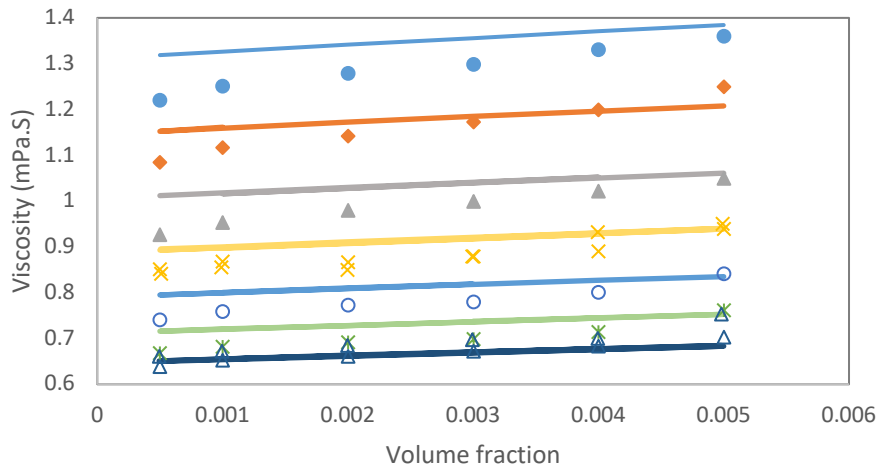


Figure 1. Effect of GQD concentrations on the viscosity of water at various temperature; (●) 283 K, (◆) 288 K, (▲) 293 K, (■) 298 K, (○) 303 K, (*) 308 K and (Δ) 313 K. (Solid line) present model in this work.

In Fig. 2, the calculated viscosities based on the present model in this work are plotted versus experimental viscosities as a graphic tool for showing the deviation. Fig. 2 shows that the scattering of the results around the symmetry line ($\mu^{cal} = \mu^{exp}$) is acceptable for present model, however, most of the points are below the symmetry line which means that the results of the present work show slightly under-correlated ($\mu^{cal} / \mu^{exp} < 1$).

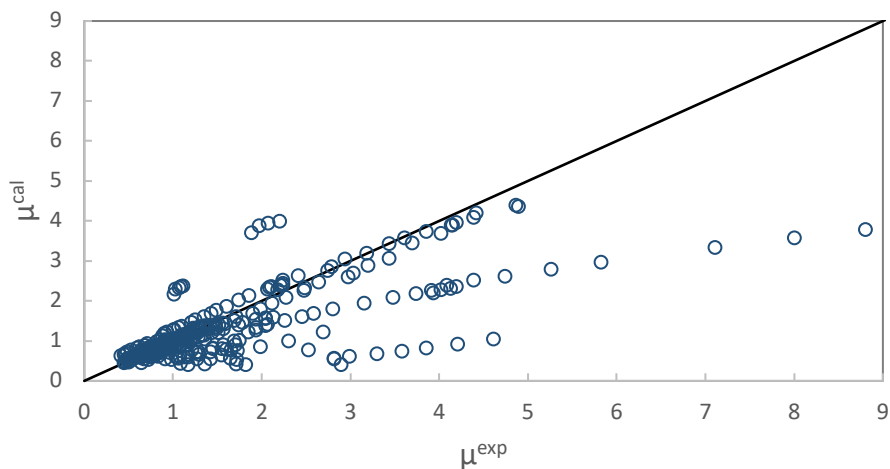


Figure 2. Calculated viscosity versus experimental viscosity for all the nanofluids.

On the other hand, Fig. 3 displays the deviation of the present model based on the percent of relative deviation respect to the volume fraction of the nanoparticles in the solutions for all binary mixtures. It can be seen from Fig. 3, high deviations occur in very low ($\phi < 0.01$) and very high ($\phi > 0.07$) concentration of the nanoparticles in the liquid phase and in the middle range of the volume fraction ($0.01 < \phi < 0.07$) the deviation is moderate.

Finally, Fig. 4 shows the viscosity deviation of the present model versus the temperature for all binary mixtures which are studied in present work. As one can see from Fig. 4 the maximum deviations for the viscosity are observed at the temperature range of 290-300 K, however, this happens for a few experimental points.

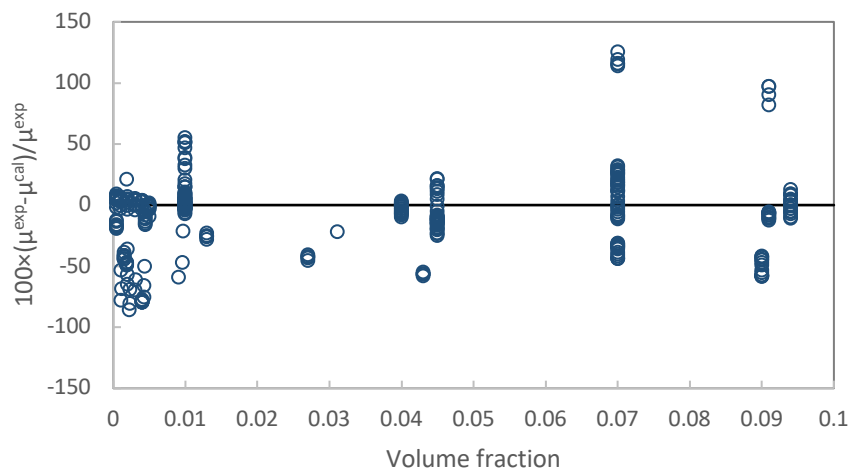


Figure 3. Percent of relative deviation versus volume fractions of nanoparticle for all of the nanofluids.

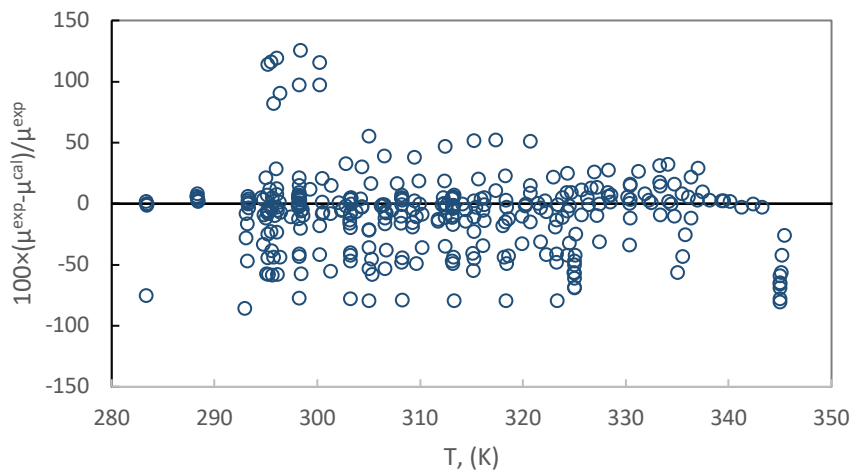


Figure 4. Percent of relative deviation versus temperature for all of the nanofluid systems.

Conclusions

In this article, a new simple empirical model was introduced for the prediction of the viscosity of nanofluids as an important thermodynamic property of nanofluids. New model is only based on the volume fraction of the nanoparticles in the liquid phase. To validate the new model, five different nanofluids with water as the base fluid and various nanoparticles (water- Al_2O_3 , water- SiO_2 , water- TiO_2 , water-GQD, water-CuO) at various temperatures and concentrations of nanoparticles were considered. Comparisons between the results of the present model with experimental data showed an acceptable overall deviation (AARD%=22.02%), as the lowest deviation (AARD%=3.79%) occur for the water-GQD system, while highest deviation (AARD% = 52.13 %) arise for the water- TiO_2 system. The results demonstrate that the high deviations are observed in low and high volume fraction range of nanoparticle. Also, an under correlated was detected in the results of the presented model. Finally, the comparison shows that the model presented in this work has the best performance than other five models.



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